

逢甲大學九十四學年度轉學生招生考試試題

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| 科目 | 線性代數 | 適用系別 | 應數系三年級 | 時間 | 八十分鐘 |
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1. (30%) Label the following statements as being true or false.
 - (a) An $m \times n$ matrix has m columns and n rows.
 - (b) If V is a vector space and W is a subset of V then W is a subspace of V .
 - (c) The intersection of any two subsets of V is a subspace of V .
 - (d) The trace of a square matrix is the product of its diagonal entries.
 - (e) Every change of coordinate matrix is invertible.
 - (f) The matrices $A, B \in M_{n \times n}(F)$ are called similar if $B = Q^{-1}AQ$ for some $Q \in M_{n \times n}(F)$.
 - (g) The function $\det : M_{2 \times 2}(F) \rightarrow F$ is a linear transformation.
 - (h) A matrix $M \in M_{n \times n}(F)$ is invertible if and only if $\det(M) \neq 0$.
 - (i) Every system of n linear equations in n unknowns can be solved by Cramer's rule.
 - (j) A matrix Q is an invertible matrix, then $\det(Q^{-1}) = [\det(Q)]^{-1}$.

2. (20%) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for \mathbb{R}^2 and $\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$. Compute $[T]_{\beta}^{\gamma}$. If $\alpha = \{(1,2), (2,3)\}$, compute $[T]_{\alpha}^{\gamma}$.

3. (10%) Find an expression for $T(x, y)$ for any (x, y) in \mathbb{R}^2 , where T is the reflection about the line $y = 2x$.

4. (20%) Using Cramer's rule to solve the system of linear equation below.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 2 \\ x_1 + x_3 &= 3 \\ x_1 + x_2 - x_3 &= 1. \end{aligned}$$

5. (20%) Find the inverse of the matrix $A = \begin{pmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{pmatrix}$.