

逢甲大學九十四學年度轉學生招生考試試題

科目	高等微積分	適用系別	應數系三年級	時間	八十分鐘
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A. 是非題(32%)

1. If $\emptyset \neq E \subseteq \mathbb{R}$ is connected, then E is an interval.
2. Let $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$ be continuous and $H \subset \mathbb{R}^n$ be compact. Then $f(H)$ is compact in \mathbb{R}^m .
3. \mathbb{Q} (the set of rational numbers) is complete.
4. \mathbb{R} (the set of real numbers) is uncountable.
5. $\bigcap_{n=1}^{\infty} (0, \frac{1}{n}) \neq \emptyset$.
6. $\{\frac{1}{n} \mid n \in \mathbb{N}\}$ is compact.
7. $\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ is open in \mathbb{R}^2 .
8. Let $\{x_n\} \subset \mathbb{R}$ be convergent. Then $\{x_n\}$ is a Cauchy sequence.

B. 計算證明題

- (1) Let $x_n = \frac{1}{n} - (-1)^n$ and $E = \{\frac{1}{n} - (-1)^n \mid n \in \mathbb{N}\}$.
 - (a) Find $\sup E$ and $\inf E$. (10%)
 - (b) Find $\overline{\lim} x_n$ and $\underline{\lim} x_n$. (10%)
- (2) Suppose that $0 < x_1 < 1$ and $x_{n+1} = 1 - \sqrt{1 - x_n}$ for $n \in \mathbb{N}$.
 - (a) Prove that $0 < x_n < 1$ holds for all $n \in \mathbb{N}$. (6%)
 - (b) Prove that $x_{n+1} < x_n$ holds for all $n \in \mathbb{N}$. (6%)
 - (c) Find $\lim_{n \rightarrow \infty} x_n$. (6%)
- (3) Determine whether $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$ exists or not. (10%)
- (4) Find $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$. (10%)
- (5) Let $f(x, y) = \begin{cases} \frac{x^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$. Find $f_x(0, 0)$ and $f_y(0, 0)$ by the definition. (10%)