

科目	工程數學	適用 系別	化工系三年級	時間	80分鐘
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※ 請務必在答案卷作答區內作答 ※

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1. (20%) A tank contains 1000 gal of water in which initially 100 lb of salt is dissolved. The inflow salty solution runs in at a rate of 10 gal/min, and each gallon contains 5 lb of dissolved salt. The mixture in the tank is kept uniform by stirring. The out flow salty solution runs out at 10 gal/min. Find the amount of salt in the tank at any time. Hint: you can assume the amount of salt in the tank at any time as  $y(t)$ .

2. (20%) solve the ODEs

(a).  $y'' - 2y' - 3y = 0$ ,  $y(0) = 2$ ,  $y'(0) = 14$

(b).  $x^2 y'' + 3xy' + y = 0$

3. (10%) find the eigenvalues and eigenvectors of the matrix.

$$A = \begin{bmatrix} -4 & 4 \\ -1.6 & 1.2 \end{bmatrix}$$

4. (20%) Solve the following initial value problems by the Laplace transform. (show the details of your work):  $y'' + y = 2 \cos t$ ,  $y(0) = 3$ ,  $y'(0) = 4$

5. (10 %) Find the eigenvalues and eigenfunctions of the following problem. (Show the details of your wor.):  $y'' + \lambda y = 0$ ,  $y(0) = y(2\pi)$ ,  $y'(0) = y'(2\pi)$

6. (20%) Solve the following problem. (Show the details of your work):

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, u(x,0) = 0, u(x,a) = f(x), u(0,y) = 0, u(b,y) = 0$$