逢甲大學96學年度碩、博士班甄試入學試題

科目	統計(含機率)	適用系所	統計與精算研究所	時間	一〇〇分鐘

※請務必在答案卷作答區內作答。

1. (15%) Prove or disprove each of the following statements (you may assume that none of the events has zero probability):
   (a) (5%) $P(A \cap B) \geqq 1 - P(A^c) - P(B^c)$.
   (b) (5%) If $P(A \mid C) = P(B \mid C)$ and $P(A \mid C^c) = P(B \mid C^c)$, then $P(A) = P(B)$.
   (c) (5%) If events A, B, and C are independent, then
   $$P(A \cup B \cup C) \geqq 1 - [1 - P(A)]\,[1 - P(B)].$$

2. (15%) A slip of paper is given to person A, who marks it with either a plus or minus sign. The probability of her writing a plus is 1/4. Person A passes the slip to person B, who may either leave it alone or change the sign before passing it to person C. Next, person C passes the slip to person D after perhaps changing the sign. Finally, person D passes it to a referee after perhaps changing the sign. It is known that B, C, and D each change the sign with probability 3/4.
   (a) (10%) What is the probability that the referee sees a plus sign on the slip?
   (b) (5%) Given that the referee saw a plus sign on the slip, what is the probability that person A originally wrote a plus sign?

3. (20%) If X and Y are independent exponentially distributed random variables with means 1/6 and 1/9 respectively,
   (a) (10%) What is $P[minimum(X, Y) > 10]$?
   (b) (10%) What is $P[X = minimum(X, Y)]$?

4. (25%) A political scientist suspects that there is a relationship between the number of promises a political candidate makes (X) and the number of promises that are fulfilled (Y) once the candidate is elected. There are 10 politicians are investigated. The data is summarized below:

$\sum X = 433,\quad \sum Y = 58 \quad \sum X^2 = 20{,}301 \quad \sum Y^2 = 406 \quad \sum XY = 2{,}376$

(a) (10%) Write down the linear regression line to see how well the "promises kept" (Y) can be estimated by the "promises made" (X).

(b) (10%) Write down the hypotheses and test the significance of the regression model. Use a level of significance of 0.05.

(c) (5%) Calculate the correlation coefficient between promises make (X) and kept (Y).

5. (15%) A portfolio of 10,000 risks produced the following numbers of claims.

No. of Claims	0	1	2	3	4+
No. of Policies	9,048	905	45	2	0

(a) (5%) Determine the mle of λ for a Poisson model.

(b) (10%) Conduct the chi-square goodness-of-fit test at α=0.05.

6. (10%) Here are the means, standard deviations, and correlations for the annual returns from three Fidelity mutual funds for the 10 years ending in February 2004. The statistics are summarized in the following.

Variable	**Fund**	**Mean return**	**Standard deviation of return**
W	Annual return on 500 Index Fund	11.12%	17.46%
X	Annual return on Investment Graded Bond Fund	6.46%	4.18%
Y	Annual return on Diversified International Fund	11.1%	15.62%
Correlations: $\rho_{WX} = -0.22$ $\rho_{WY} = 0.56$ $\rho_{XY} = -0.12$			

A portfolio of 80% 500 Index and 20% Diversified International will be set. Find the mean and standard deviation of returns on this portfolio.

t distribution critical value $\Pr(t > 3.078\ , df = 1) = 0.100$

Upper-tail probability p

df	0.100	0.050	0.025	0.010	0.005	0.0005
1	3.078	6.314	12.706	31.821	63.66	636.62
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587

Chi-square distribution critical values $\Pr(\chi^2 > 6.635, df = 1) = 0.01$

Upper-tail probability p

df	0.25	0.2	0.15	0.10	0.05	0.025	0.02	0.01	0.005	0.003
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98
3	4.11	4.64	5.32	6.25	7.81	11.14	9.84	11.34	12.84	14.32
4	5.39	5.99	6.74	7.78	9.49	12.83	11.67	13.28	14.86	16.42